UNIT- IV

RELATIVITY

Q. NO.*

Introduction

The first theory in physics was mechanics developed by Newton and it is called classical ~~th~~ or Newtonian mechanics. Actually the physics developed before the 20^{th} century is called classical physics.

Classical mechanics presumes the space the space to be homogeneous in all its parts and also isotropic. It means that the properties of space are identical at all points and in all directions at each point. classical mechanics further supposes the existance of an absolute space, which is absolutely motionless and irrelavant to the existance of any body. According to Newton " absolute space, in its own nature, without regard to anything external, remains always similar and immovable."

In classical mechanics, time is understood as a measure of absolute duration, which exists irrespective of physical bodies.

The development of 'theory of relativity' by Einstein in 1905 revolutionized the old concepts. It discards the absolute motion through space and deals with objects or observer moving with relative velocities w.r.t. each other. This theory is divided into two parts

(i) special theory

(ii) General theory

The special theory of relativity deals with objects and systems which are either moving at a constant speed w.r.t. one another or are at rest.

The general Theory deals with objects or system which are speeding up or slowing down with respect to one another

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Frame of Reference

Rest and motion are relative terms. To define motion, the observer must define a frame of reference relative to which the motion is considered.

A body in motion can be located with reference to some co-ordinate system called the frame of reference. If the co-ordinates of all the points of a body remain unchanged with time and w.r.t the frame of reference, the body is said to be at rest. If the co-ordinates of any point of the body change with time and w.r.t the frame of reference, the body is said to be in motion.

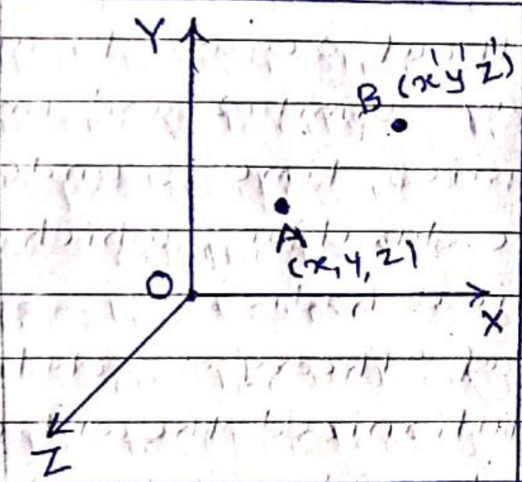

Fig. 1

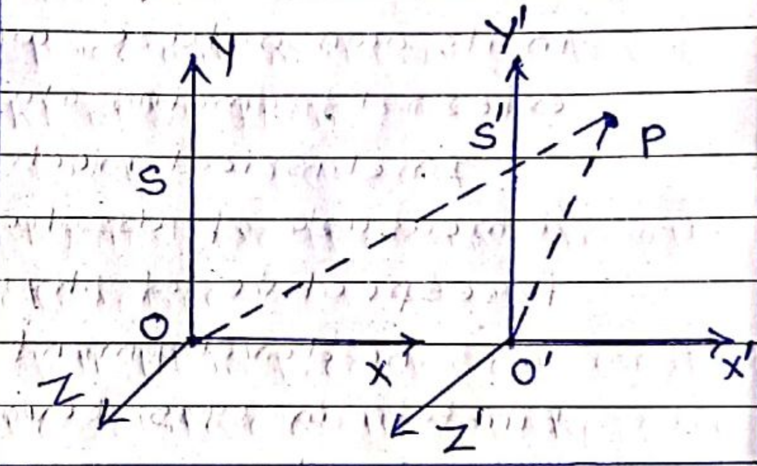

Fig 2

Consider a body P is at the point A. Its co-ordinates are (x, y, z) with respect to the frame of reference (Fig. 1). If the body P always remains at A, it will be at rest w.r.t the frame of reference. If another body Q is initially at A and after some time it is at B(x' y' z'), it is in motion w.r.t the frame of reference.

Now consider two frame of reference S and S', as shown in Fig. 2

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The observer O considers the motion of P w.r.t the frame of reference S, and the observer O' w.r.t. the frame of reference S'.

If O and O' are at rest w.r.t. each other, they will observe the same motion of P. If O and O' are in relative motion, their observation of motion will be different.

The frame of reference is selected in such a way that the laws of nature may become fundamentally simpler in that frame of reference. There are two types of frames of reference

(i) Inertial or unaccelerated frames

(ii) Non-inertial or accelerated frames

(i) Inertial or unaccelerated frames

A frame of reference is said to be inertial when bodies in this frame obey Newton's law of inertia and other laws of Newtonian mechanics. In this frame, a body not acted by external force, is at rest or moves with constant velocity. In this frame of reference no acceleration is observed for a particle free of any force.

Inertial frame of reference is also called the Galilean or Newtonian frame of reference.

In inertial fram of reference the laws of physics will be same for all observers.

(ii) Non-inertial or accelerated frames.

A frame of reference is said to be non-inertial frame when a body, not acted upon by an external force, is accelerated. ~~motion~~ In this frame, the

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Newton's laws are not valid.

* Postulates of special Theory of Relativity

Einstein in 1905 announced the special theory of relativity and in 1915 proposed the general theory of relativity. The special theory deals with the problems in which one frame of reference moves with a constant linear velocity relative to another frame of reference.

The general theory of relativity deals with problems in which one frame of reference is accelerated w.r.t another frame of reference.

Einstein assumed that all observers will notice that their motion in space will make no difference in the velocity of light w.r.t them.

Postulates

(i) The laws of physics are the same in all inertial frames of reference.

(ii) The velocity of light in free space is constant. It is independent of the relative motion of the source and the observer.

~~Eis~~ Einstein proved the following facts based on his theory of relativity.

Let v be the velocity of the spaceship w.r.t a given frame of reference. where the observer makes the observations

(a) All clocks on the spaceship will go slow by a factor

$$\sqrt{1-\frac{v^2}{c^2}}$$

(b) All objects on the spaceship will have contracted to a length by a factor $\sqrt{1-\frac{v^2}{c^2}}$

Q.NO(c) The mass of the spaceship increases by a factor

$$\left[\frac{1}{\sqrt{1-v^2/c^2}}\right]$$

(d) Mass and energy are interconvertible

$$E = mc^2$$

(e) The speed of material object can never exceeds the velocity of light

(f) If two objects A and B are moving with velocities u and v with respect to each other along the x-axis, the relative velocity of A with respect to B is given by

$$V_x = \frac{u-v}{1-\frac{uv}{c^2}}$$

where u and v are comparable with the value of c

* Galilean Transformation

Let S and S' be two inertial frames as shown in below fig 1.

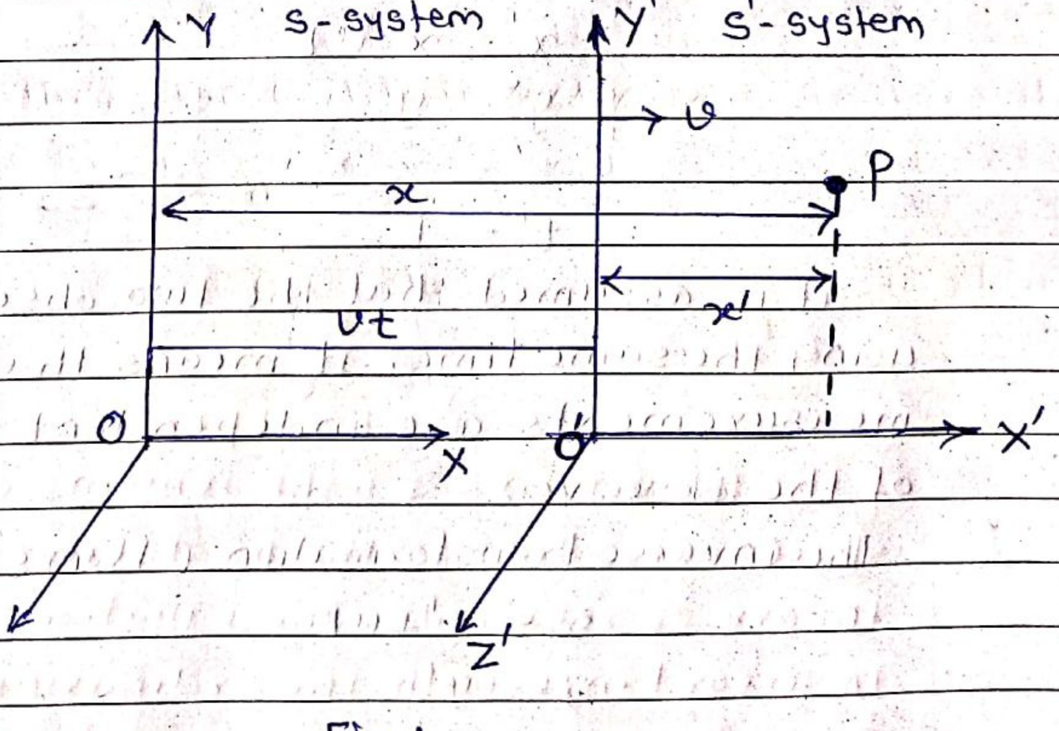

Fig.1

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Consider two identical frames of reference S & S'. The observer O is in the reference frame S and O' in reference frame S'. The observer O' moves with a uniform velocity v relative to O along the x-axis. At time $t=0$, O and O' are coincident. After time t, $OO' = vt$

Let some event occurs at the point P. The observer O in frame S determines the position of the event by the co-ordinates x, y, z. The observer O' in frame S' determines the position of the event by the co-ordinates x', y', z'. There is no relative motion between S and S' along the axes of Y and Z. Hence we have $y = y'$ and $z = z'$

So, the distance moved by S' in the positive x-direction in time $t = vt$. Hence the x-coordinates of the two frames differ by vt.

$$\therefore \quad x' = x - vt$$

Then the transformation equations from S to S' are given by

$$x' = x - vt \quad \text{———} \quad (1)$$

$$y' = y \quad \text{———} \quad (2)$$

$$z' = z \quad \text{———} \quad (3)$$

$$t' = t \quad \text{———} \quad (4)$$

It is assumed that the two observers are using the same time. It means that the time measurements are independent of the motion of the observer

The inverse transformation (from S' to S) will be

$$x = x' + vt$$

$$y = y'$$

$$z = z'$$

$$t = t'$$

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(I) Galilean Velocity Transformations

The velocity of the particle assigned by the observer O will be (u_x, u_y, u_z) and by the observer O' will be (u_x', u_y', u_z').

The relationship between (u_x, u_y, u_z) and (u_x', u_y', u_z') is obtained from the time differentiation of the Galilean co-ordinate transformations.

Thus from $x' = x - vt$

$$u_x' = \frac{dx'}{dt} = \frac{dx}{dt} - v = u_x - v$$

$$\therefore \; u_x' = u_x - v, \quad u_y' = u_y, \quad u_z' = u_z$$

(II) Galilean Acceleration Transformations

The acceleration of particle is the time derivative of its velocity i.e. $a_x = du_x/dt$

we have seen above that $u_x' = u_x - v$

$$\therefore \; a_x' = \frac{du_x'}{dt} = \frac{du_x}{dt} - 0$$

$$\therefore \; a_x' = \frac{du_x}{dt} \qquad \therefore \; a_x' = a_x$$

Altogether, the Galilean ~~velocity~~ acceleration transformations are

$$a_x' = a_x$$

$$a_y' = a_y$$

$$a_z' = a_z$$

Q. NO. * **Lorentz Transformations**

Consider two observers O and O' in two systems S and S'. System S' is moving with a constant velocity v relative to system S along the positive X-axis. Suppose we make measurements of time from the instant when the origins of S and S' just coincide i.e. $t=0$ when O and O' coincide. Suppose a light pulse is emitted when O and O' coincide. The light pulse produced at $t=0$ will spread out as a growing sphere. The radius of the wave-front produced in this way will grow grow with speed c. After a time t, the observer O will note that the light has reached a point $P(x, y, z)$ as shown in below Fig. 1.

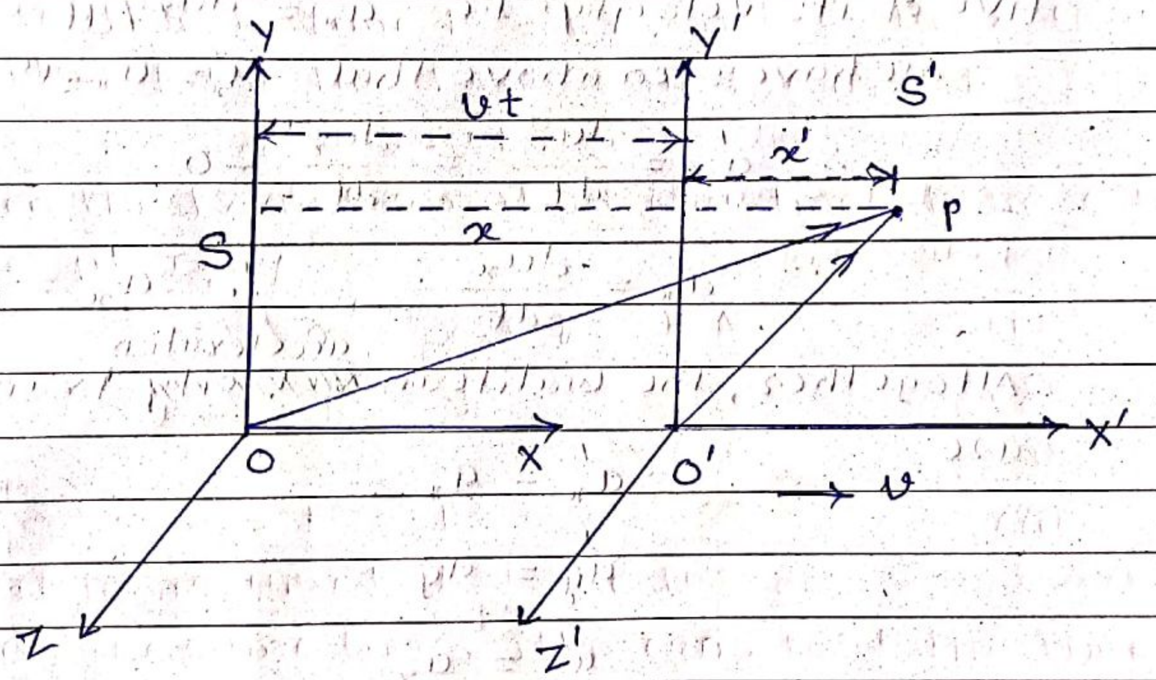

Fig 1

The velocity of light remains constant in free space, according to the postulates of the special theory of relativity. The equation $x' = x - vt$ is in accordance with the ordinary laws of mechanics. The new transformation for the X-coordinate

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must be similar to this equation when the value of v is extremely small as compared to the velocity of light c. The simplest possible form of this eq^n ~~is~~ can be

$$x' = k(x - vt) \quad \text{------} (1)$$

Here k is constant which depends only on the value of v and does not depend upon the values of x and t.

According to the first postulate of the special theory of relativity, observations made in the frame of reference S' must be identicle to those made in S, except for a change in the sign of v and having the same value for the constant of proportionality k

$$\therefore \quad x = k(x' + vt') \quad \text{----------} (2)$$

As the relative motion of S and S' is confined to only x

$$y' = y \quad \text{------------} (3)$$

$$z' = z \quad \text{----------} (4)$$

But this equality does not hold good for t and t'

Put the value of x' from eq^n (1) in eq^n (2). This gives

$$x = k[k(x - vt) + vt']$$

$$\therefore \quad x = k^2(x - vt) + kvt' \quad \text{-----} (5)$$

$$\therefore \quad kvt' = x - k^2(x - vt)$$

$$\therefore \quad t' = \frac{x - k^2x + k^2vt}{kv}$$

$$t' = \frac{k^2vt}{kv} + \frac{x(1-k^2)}{kv}$$

$$\therefore \quad t' = kt + \left(\frac{1-k^2}{kv}\right)x \quad \text{-----} (6)$$

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To find the value of k, consider two frames of reference S and S'. The spaceship in reference frame S measures the time t and the spaceship in reference frame B measures the time t', for a flash of light. The x-coordinates for both the ships will be

$$x = ct \quad \text{-------- (7)}$$

and

$$x' = ct' \quad \text{-------- (8)}$$

The value of c must remain constant in both the frames of reference.

So put the value of x' and t' in eqⁿ (8)

$$\therefore \; k(x - vt) = c\left[kt + \left(\frac{1-k^2}{kv}\right)x\right]$$

$$\therefore \; kx - kvt = ckt + cx\left(\frac{1-k^2}{kv}\right)$$

$$\therefore \; kx - cx\left(\frac{1-k^2}{kv}\right) = ckt + kvt$$

$$\therefore \; x\left[k - c\left(\frac{1-k^2}{kv}\right)\right] = ckt\left(1 + \frac{v}{c}\right)$$

$$\therefore \; x = \frac{ckt\left(1+\frac{v}{c}\right)}{\left[k - c\left(\frac{1-k^2}{kv}\right)\right]} = \frac{ckt\,(1+v/c)}{k\left[1 - c\left(\frac{1-k^2}{k^2 v}\right)\right]}$$

$$\therefore \; x = ct\left[\frac{\left(1+\frac{v}{c}\right)}{1 - \frac{c}{v}\left(\frac{1-k^2}{k^2}\right)}\right]$$

$$\therefore \; x = ct\left[\frac{\left(1+\frac{v}{c}\right)}{1 - \left(\frac{c}{v}\right)\left(\frac{1}{k^2} - 1\right)}\right] \quad \text{--- (9)}$$

Q. NO. comparing eqⁿ (7) and (9) we get

$$1 = \left[\frac{\left(1+\frac{v}{c}\right)}{1-\left(\frac{c}{v}\right)\left(\frac{1}{k^2}-1\right)}\right]$$

$$\therefore 1-\left(\frac{c}{v}\right)\left(\frac{1}{k^2}-1\right) = 1+\frac{v}{c}$$

$$\therefore -\frac{c}{v}\left(\frac{1}{k^2}-1\right) = \frac{v}{c}$$

$$\therefore \frac{c}{v}\left(1-\frac{1}{k^2}\right) = \frac{v}{c}$$

$$\therefore 1-\frac{1}{k^2} = \frac{v^2}{c^2}$$

$$\therefore \frac{1}{k^2} = 1-\frac{v^2}{c^2}$$

$$\therefore k^2 = \frac{1}{\left(1-\frac{v^2}{c^2}\right)}$$

$$\therefore k = \frac{1}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}} \quad \text{------ (10)}$$

The value of k is in accordance with the experimental result. when the value of v is extremely small in comparision to c, the value of k is equal to 1.

substituting the value of k in equations (1), (3), (4) and (6), we get

$$x' = k(x-vt)$$

$$\therefore x' = \frac{(x-vt)}{\sqrt{1-v^2/c^2}} \quad \text{------ (11)}$$

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$$y' = y \quad \text{-----------} (12)$$

$$z' = z \quad \text{-----------} (13)$$

$$t' = kt\left(\frac{1-k^2}{kv}\right)x = kt + \frac{x}{kv} - k\left(\frac{x}{v}\right)$$

$$\therefore t' = kt + \frac{x}{v}\left(\frac{1}{k} - k\right) = kt + \frac{x}{v}k\left(\frac{1}{k^2} - 1\right)$$

$$t' = k\left[t + \frac{x}{v}\left(\frac{1}{k^2} - 1\right)\right]$$

$$t' = \frac{t + \frac{x}{v}\left[1 - \frac{v^2}{c^2} - 1\right]}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\therefore t' = \frac{t + vx/c^2}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{-----------} (14)$$

Equations (11), (12), (13) and (14) are called the Lorentz transformations.

The inverse Lorentz transformation equations are obtained by interchanging the coordinates and replacing v by $-v$ in the above

$$x = \frac{x' + vt'}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{-----------} (15)$$

$$y = y' \quad \text{-----------} (16)$$

$$z = z' \quad \text{-----------} (17)$$

and $$t = \frac{t' + \left(vx'/c^2\right)}{\sqrt{1 - V^2/c^2}} \quad \text{-----------} (18)$$

These equations convert measurements made in frame S' into those in frame S.

Q. NO.* **Length contraction (Lorentz-Fitzgerald contraction)**

Consider two co-ordinate system S and S' with their X-axis coinciding at time $t=0$. S' is moving with a uniform relative speed v w.r.t. S' in the positive X-direction. Let a rod AB is at rest relative to S' is shown in Fig 1.

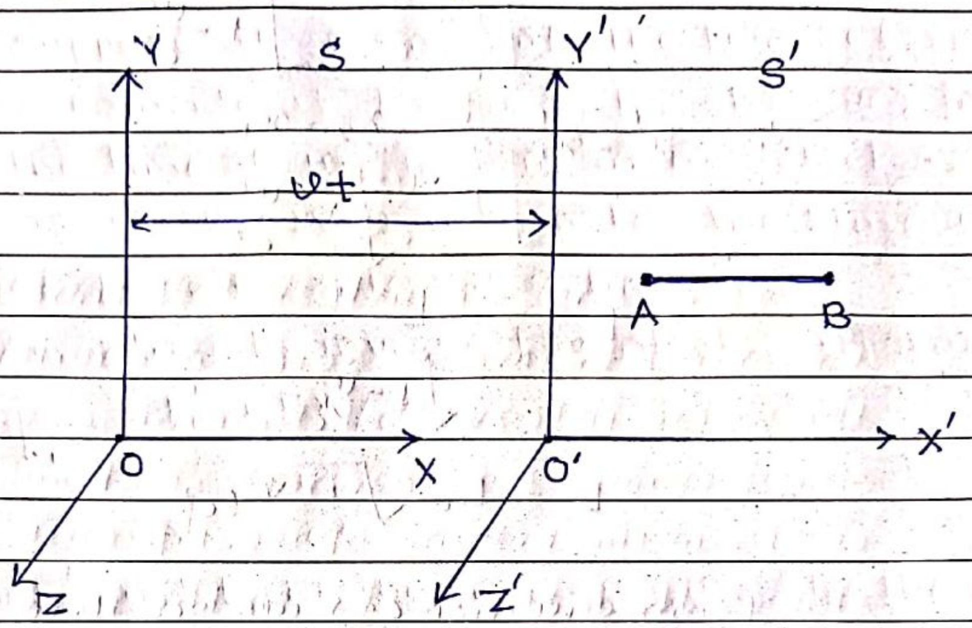

Fig.1

Let x_1' and x_2' be the co-ordinates of the ends of the rod at any instant of time in S'. Then

$$L_0 = x_2' - x_1' \quad \text{------- (1)}$$

since the rod is at rest in the frame S'.

Similarly, let x_1 and x_2 be the co-ordinates of the ends of the rod at the same instant of time in S, is

$$L = x_2 - x_1 \quad \text{--------- (2)}$$

L is the length of the rod, measured relative to S.

According to Lorentz transformations,

$$x_2' = \frac{x_2 - vt}{\sqrt{1 - v^2/c^2}} \quad \text{---- (3)}$$

$$x_1' = \frac{x_1 - vt}{\sqrt{1 - v^2/c^2}} \quad \text{---- (4)}$$

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subtracting equation (4) from (3)

$$x_2' - x_1' = \frac{x_2 - vt}{\sqrt{1-\frac{v^2}{c^2}}} - \frac{x_1 - vt}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$= \frac{x_2 - vt - x_1 + vt}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\therefore \quad x_2' - x_1' = \frac{x_2 - x_1}{\sqrt{1-\frac{v^2}{c^2}}}$$

or

$$L_0 = \frac{L}{\sqrt{1-v^2/c^2}}$$

$$\therefore \quad L = L_0\sqrt{1-v^2/c^2} \qquad \text{------(5)}$$

From equation (5) we see that $L < L_0$. Therefore, to the observer in S it would appear that the length of the rod is contracted by the factor $\sqrt{1-v^2/c^2}$.

Notes:-

(1) The proper length of an object is the length determined by an observer at rest w.r.t. object.

(2) The shortning or contraction in the length of an object along its direction of motion is known as Lorentz-Fitzgerald contraction. There is no contraction in a direction perpendicular to the direction of motion

(3) The contraction becomes appreciable only when $V \approx C$.

Q. NO. * **Time Dilation**

According to the special theory of relativity, time intervals are also affected by the relative motion between two frames of reference.

Suppose S and S' are two frames of reference. Let S' is moving with a velocity v along the x-axis.

Suppose at any instant, the two reference frames coincide at $t = t' = 0$. The observer in S' notes the time at any instant in his clock as t_1' whereas an observer in S notes the time as t_1.

~~Let t_0 be the interval of time~~

According to inverse Lorentz transformations

$$t_1 = \frac{t_1' + vx'/c^2}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{------ (1)}$$

Let t_2' and t_2 be the time measured by the observers in S' and S respectively at the same instant

so

$$t_2 = \frac{t_2' + \frac{vx'}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{------ (2)}$$

Let t_0 be the interval of time measured by an observer in S' and t the interval of time measured by an observer in S

$$\therefore t_0 = t_2' - t_1' \quad \text{------ (3)}$$

and

$$t = t_2 - t_1 \quad \text{------ (4)}$$

$$\therefore t = \frac{t_2' + \frac{vx'}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} - \frac{t_1' + \frac{vx'}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

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$$\therefore t = \frac{t_2' + \frac{vx'}{c^2} - t_1' - \frac{vx'}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$t = \frac{t_2' - t_1'}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\therefore t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}} \quad \text{———— (5)}$$

It means, an interval of time observed in a moving frame of reference will be less than the same interval of time observed in a stationary frame of reference, It shows that the clocks in the moving spaceship will appear to go slower than the clocks on the surface of the earth. It is called time dilation.

The twin Paradox

Consider two exactly identical ~~twin~~ twin brothers. Let one of the twins go to a long journey at a high speed in a rocket and the other stay behind on the earth. The clock in the moving rocket will appear to go slower than the clock on the surface of the earth. Therefore, when he returns back to the earth, he will be find himself younger than the twin who stayed ~~on~~ behind on the earth.

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Q. NO. * **Velocity Addition**

Suppose the system S' moves with a uniform velocity v relative to the system S. Suppose a particle is moving in the common direction of the x and x' axes. Let its velocity, as measured by an observer in the system S, be u and as measured by an observer in S' be u'.

Then we have, $u = \frac{dx}{dt}$ and $u' = \frac{dx'}{dt'}$

Using inverse Lorentz transformations, we have

$$x = \frac{x' + vt'}{\sqrt{1 - v^2/c^2}} \quad \text{and} \quad t = \frac{t' + vx'/c^2}{\sqrt{1 - v^2/c^2}}$$

Differentiating,

$$dx = \frac{dx' + v\,dt'}{\sqrt{1 - v^2/c^2}} \quad \text{and} \quad dt = \frac{dt' + v\,dx'/c^2}{\sqrt{1 - v^2/c^2}}$$

Thus

$$\frac{dx}{dt} = \frac{\dfrac{dx' + v\,dt'}{\sqrt{1 - v^2/c^2}}}{\dfrac{dt' + v\,dx'/c^2}{\sqrt{1 - v^2/c^2}}}$$

$$\frac{dx}{dt} = \frac{dx' + v\,dt'}{dt' + \dfrac{v\,dx'}{c^2}} = \frac{dx' + v\,dt'}{dt' + \dfrac{v}{c^2}dx'}$$

Multiplying ~~as~~ by dt' to the numerator and denominator

$$\frac{dx}{dt} = \frac{\left(\dfrac{dx'}{dt'}\right) + v}{1 + \dfrac{v}{c^2}\left(\dfrac{dx'}{dt'}\right)}$$

But $\frac{dx'}{dt'} = u'$

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$$\therefore \quad u = \frac{u' + v}{1 + \frac{u'v}{c^2}}$$

If we put $u' = c$ and $v = c$ i.e photon is moving with a velocity c in frame s' and s' is moving with velocity c relative to S, then

$$u = \frac{c + c}{1 + (c^2/c^2)} = \frac{2c}{1+1} = \frac{2c}{2}$$

$$\therefore \quad u = c$$

Thus, the addition of any velocity to the velocity of light c merely reproduces the velocity of light. Hence, the velocity of light is the maximum attainable velocity.

Notes:-

<1> This law of addition of velocities applies only when the two velocities are in the same direction

<2> If $u << c$, we get the classical equation

<3> we can express the velocity u' in terms of u and v

$$u' = \frac{u - v}{1 - \frac{uv}{c^2}}$$

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Q.No * Relativity of Mass (Variation of mass with velocity)

Consider two systems S and S'. S' is moving with a constant velocity v relative to the system S, in the positive X-direction as shown in below fig 1

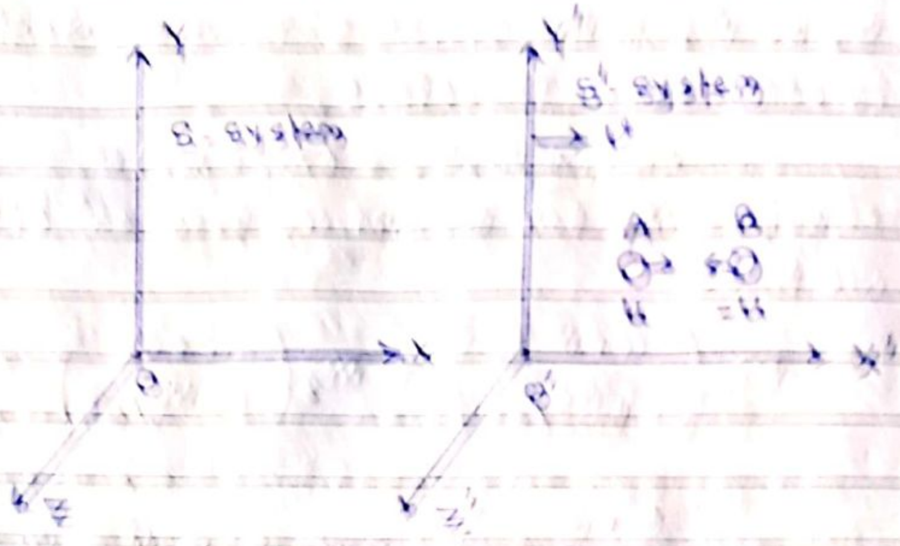

Fig 1

Suppose in system S', two exactly similar elastic balls A and B approach each other at equal speed (ie u and -u). Let the mass of each ball be m in S'. They collide with each other and after collision coalesce into one body. According to the law of conservation of momentum

Momentum of ball A + momentum of ball B = momentum of coalesced mass

or $mu + (-mu)$ = momentum of coalesced mass $= 0$

Thus coalesced mass must be at rest in S'.

Now consider the collision with reference to the system S.

Let u_1 and u_2 be the velocities of the balls relative to S. Then

$$u_1 = \frac{u+v}{1+uv/c^2} \quad \text{---------- (1)}$$

and

$$u_2 = \frac{-u+v}{1-uv/c^2} \quad \text{---------- (2)}$$

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After collision, velocity of the coalesced mass v relative to the system S.

Let mass of the ball A travelling with velocity u_1 be m_1 and mass of the ball B travelling with velocity u_2 be m_2 in the system S. Total momentum of the balls is conserved. Therefore,

$$m_1u_1 + m_2u_2 = (m_1+m_2)v \quad \text{------} (3)$$

Substituting for u_1 and u_2 from eq^n (1) and (2), we have

$$m_1\left(\frac{u+v}{1+uv/c^2}\right) + m_2\left(\frac{-u+v}{1-uv/c^2}\right) = (m_1+m_2)v$$

$$\therefore m_1\left(\frac{u+v}{1+uv/c^2}\right) - m_1v = m_2v - m_2\left(\frac{-u+v}{1-uv/c^2}\right)$$

$$\therefore m_1\left[\frac{u+v}{1+uv/c^2} - v\right] = m_2\left[v - \frac{-u+v}{1-uv/c^2}\right]$$

$$\therefore m_1\left[\frac{u+v-v-uv^2/c^2}{1+uv^2/c^2}\right] = m_2\left[\frac{v-uv/c^2+u-v}{1-uv/c^2}\right]$$

$$\therefore m_1\left[\frac{u-uv^2/c^2}{1+uv^2/c^2}\right] = m_2\left[\frac{u-uv^2/c^2}{1-uv/c^2}\right]$$

$$\therefore m_1\left[\frac{u\,(1-uv/c^2)}{1+uv^2/c^2}\right] = m_2\left[\frac{u\,(1-uv/c^2)}{1-uv/c^2}\right]$$

$$\text{or} \quad \frac{m_1}{m_2} = \frac{1+uv/c^2}{1-uv/c^2} \quad \text{------} (4)$$

Also, $$1-\frac{u_1^2}{c^2} = 1 - \frac{\left[(u+v)/c\right]^2}{\left(1+\frac{uv}{c^2}\right)^2}$$

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$$= \frac{\left(1+\frac{uv}{c^2}\right)^2 - \left(\frac{u+v}{c}\right)^2}{\left(1+\frac{uv}{c^2}\right)^2}$$

$$= \frac{1+\frac{u^2v^2}{c^4}+\frac{2uv}{c^2}-\frac{u^2}{c^2}-\frac{v^2}{c^2}-\frac{2uv}{c^2}}{\left(1+\frac{uv}{c^2}\right)^2}$$

$$= \frac{1+\frac{u^2v^2}{c^4}-\frac{u^2}{c^2}-\frac{v^2}{c^2}}{\left(1+\frac{uv}{c^2}\right)^2}$$

$$= \frac{\left(1-\frac{u^2}{c^2}\right)-\frac{v^2}{c^2}\left(1-\frac{u^2}{c^2}\right)}{\left(1+\frac{uv}{c^2}\right)^2}$$

$$\therefore 1-\frac{u_1^2}{c^2} = \frac{\left(1-\frac{u^2}{c^2}\right)\left(1-\frac{v^2}{c^2}\right)}{\left(1+\frac{uv}{c^2}\right)^2} \quad \text{------(5)}$$

Similarly,

$$1-\frac{u_2^2}{c^2} = \frac{\left(1-\frac{u^2}{c^2}\right)\left(1-\frac{v^2}{c^2}\right)}{\left(1-\frac{uv}{c^2}\right)^2} \quad \text{------(6)}$$

Dividing eq^n (6) by (5)

$$\frac{1-\frac{u_1^2}{c^2}}{1-\frac{u_2^2}{c^2}} = \frac{\left(1+\frac{uv}{c^2}\right)^2}{\left(1-\frac{uv}{c^2}\right)^2}$$

or

$$\frac{\sqrt{1-u_1^2/c^2}}{\sqrt{1-u_2^2/c^2}} = \frac{\left(1+\frac{uv}{c^2}\right)}{\left(1-uv/c^2\right)} \quad \text{------(7)}$$

Q. NO.

From eqⁿ (7) and (4)

$$\frac{m_1}{m_2} = \frac{\sqrt{1-u_2^2/c^2}}{\sqrt{1-u_1^2/c^2}}$$

or $$m_1\sqrt{1-\frac{u_1^2}{c^2}} = m_2\sqrt{1-\frac{u_2^2}{c^2}} \quad \text{------ (8)}$$

Since the L.H.S and RHS of equations (8) are independant of one another, the above result can be true only if each is a constant. Therefore

$$m_1\sqrt{1-\frac{u_1^2}{c^2}} = m_2\sqrt{1-\frac{u_2^2}{c^2}} = m_0$$

The constant denoted by m_0 is called the rest mass of the body and corresponds to zero velocity

Thus $$m_1 = \frac{m_0}{\sqrt{1-\frac{u_1^2}{c^2}}} \quad \text{------ (9)}$$

In general, if m denotes the mass of a body when it is moving with a velocity v then

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \quad \text{---------- (10)}$$

This is the relativistic formula for the variation of mass with velocity.

If we put $v \to c$ in equation (10), we have $m \to \infty$ ie an object travelling at the speed of light would have infinite mass. Thus eq. (10) shows that no material body can have a velocity equal to or greater than the velocity of light

Q. NO. * **Mass Energy Relation**

Force is defined as rate of change of momentum i.e

$$F = \frac{d}{dt}(mv) \quad \text{------------} (1)$$

According to the theory of relativity, both mass and velocity are ~~as~~ variable, Therefore

$$\therefore F = \frac{d}{dt}(mv) = m\frac{dv}{dt} + v\frac{dm}{dt} \quad \text{-----} (2)$$

Let the force F displaces the body through a distance dx.

Then, the increase in the kinetic energy (dE_k) of the body is equal to the work done ($F.dx$)

Hence, $dE_k = F dx = m\frac{dv}{dt}dx + v\frac{dm}{dt}dx$

But $\frac{dx}{dt} = v$

$$\therefore dE_k = mv\,dv + v^2 dm \quad \text{-----} (3)$$

According to the law of variation of mass with velocity,

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \quad \text{------} (4)$$

squaring both sides of eqn (4)

$$m^2 = \frac{m_0^2}{\left(1-\frac{v^2}{c^2}\right)} = \frac{m_0^2}{\frac{c^2-v^2}{c^2}}$$

$$\therefore m^2\left(\frac{c^2-v^2}{c^2}\right) = m_0^2$$

$$m^2(c^2-v^2) = m_0^2 c^2$$

Q. NO.

$$\therefore\ m^2c^2 - m^2v^2 = m_0^2c^2$$

or $$m^2c^2 = m_0^2c^2 + m^2v^2 \quad \text{------ (5)}$$

Differentiating eqⁿ (5)

$$c^2\, 2m\,dm = 0 + m^2\, 2v\,dv + 2m\,dm\, v^2$$

$$\therefore\ c^2 dm = mv\,dv + v^2 dm \quad \text{------ (6)}$$

From eqⁿ (3) $mv\,dv + v^2 dm = dE_K$

$$\therefore\ dE_K = c^2 dm \quad \text{------ (7)}$$

Thus, a change in K.E. dE_K is directly proportional to a change in mass dm.

When a body is at rest, its velocity is zero, (KE = 0) and $m = m_0$, when its velocity is v, its mass becomes m. Therefore, integrating eq. (7)

$$E_K = \int_0^{E_K} dE_K = c^2 \int_{m_0}^{m} dm = c^2(m - m_0)$$

$$\therefore\ E_K = mc^2 - m_0c^2 \quad \text{------ (8)}$$

This is the relativistic formula for K.E.

When the body is at rest, the internal energy stored in the body is m_0c^2. This m_0c^2 is called the rest mass energy. The total energy (E) of the body is the sum of K.E. (E_K) and rest mass energy (m_0c^2)

$$\therefore\ E = E_K + m_0c^2 = (mc^2 - m_0c^2) + m_0c^2$$
$$= mc^2 - m_0c^2 + m_0c^2$$

$$\therefore\ \boxed{E = mc^2} \quad \text{------ (9)}$$

This is Einstein's mass energy relation.

This relation indicates that mass may appear as energy and energy as mass.