CLASSICAL STATISTICS & QUANTUM STATISTICS

Position Space

Consider a system consisting of N particles distributed in a given volume V. If the system is static, all particles will remain fixed at various points in space. To completely specify the position of any particle in the three dimensional space, we must know the values of its three cartesian co-ordinates x, y and z which are mutually perpendicular to each other. As there are N particles it ~~gives~~ has 3N co-ordinates which gives complete information about such static system.

"The three-dimentional space in which the location of a particle is completely given by the three position co-ordinates, is known as position space."

A small element in position space is denoted by volume element dV and is expressed as

$$\boxed{dV = dx\,dy\,dz}$$

Momentum Space

If the system is dynamic, its particles move about with various velocities and hence possess momenta. A complete specification of such a system can not be described in terms of position co-ordinates only. For a dynamic system, we must specify the three components of momentum with the help of three velocity components v_x, v_y and v_z in addition to the three position co-ordinates x, y and z.

If m is the mass of the particle moving with a velocity v, its momentum $p = mv$

The three components of momentum are

$$P_x = mv_x, \quad P_y = mv_y \quad \text{and} \quad P_z = mv_z$$

"The three dimentional space in which momentum of a particle is completely specified by the three mutually perpendicular momentum co-ordinates p_x, p_y and p_z is known as momentum space".

$$\boxed{\text{Momentum space} = dp_x\, dp_y\, dp_z}$$

* Phase space

"A combination of position space and momentum space is known as phase space."

The phase space has six dimensions i.e. three position co-ordinates and three momentum co-ordinates, all mutually ⊥r to each other. The position of a particle in phase space is specified by a point with six co-ordinates x, y, z, p_x, p_y, p_z.

A small element in phase space is denoted by $d\tau$ and is given by

$$\boxed{d\tau = (dx\,dy\,dz)(dp_x\,dp_y\,dp_z)}$$

Three Kinds of Particles

The assemblies consist of three kinds of particles:

<1> Identical but distinguishable particle without any spin. The molecules of a gas are the particles of this kind. In quantum terms, the wave functions of the particles overlap to a negligible extent. The Maxwell-Boltzmann distribution function holds for such particles.

<2> Identical and indistinguishable particles of zero or integral spin. These particles can not be distinguished one from another because their wave functions overlap. Such particles are called bosons and do not obey the Pauli exclusion principle. The Bose-Einstein distribution function holds for them. The helium atoms at low temperature and the photons are the particles

of this kind and we shall use Bose-Einstein statistics to account for the spectrum of radiation from a black body.

(3) Identical and indistinguishable particles with odd half-integral spin ($1/2, 3/2, 5/2, \ldots$). Their wave functions overlap on each other. Such particles are called fermions or fermi particles, obeying the pauli exclusion principle. The Fermi-Dirac distribution function holds for them. Electrons, protons, neutrons etc are the examples of this kind. We shall use Fermi-Dirac statistics to study the behaviour of the free electrons in a metal that are responsible for its ability to conduct electric current.

* Maxwell-Boltzmann Distribution Law -

Let N be the total number of particles. Suppose the particles are distributed among k energy levels $E_1, E_2, \ldots, E_i, \ldots E_K$. Let $n_1, n_2, \ldots, n_i, \ldots n_k$ be the number of independent quantum states associated with the energy levels. According to the statistical basic postulates

$$N = n_1 + n_2 + \cdots + n_i + \cdots + n_k = \sum_i n_i \quad \text{—(1)}$$

and since the force of interaction between particles are negligible, we have

$$U = n_1 E_1 + n_2 E_2 + \cdots + n_i E_i + \cdots + n_k E_k = \sum_i n_i E_i \quad \text{——(2)}$$

Taking differential of eqn (1) and (2) we have

$$dN = \sum_i dn_i = 0 \quad \text{———— (3)}$$

and

$$dU = \sum_i E_i dn_i = 0 \quad \text{———— (4)}$$

The set of occupation numbers $n_1, n_2, \ldots$ is a possible mode of distribution of the particles among the

the energy levels, and it determines a macrostate of the system.

out of N number of particles the number of ways of choosing the first particle = N. When the first particle has been choosen, the number of ways of choosing the second particle = N-1.

Similarly, the number of ways of choosing the third, fourth, --- n_1^{th} particles are respectively (N-2), (N-3), --- $(N-n_1+1)$.

Therefore, the number of ways of choosing n_1 particles from N distinguishable particles is

$$= N(N-1)(N-2)\cdots(N-n_1+1)$$

$$= \frac{N(N-1)(N-2)\cdots(N-n_1+1)(N-n_1)!}{(N-n_1)!}$$

$$= \frac{N!}{(N-n_1)!}$$

But this number also includes the $n_1!$ arrangement of n_1 particles among themselves. The number of meaningful ways of choosing n_1 particles for the first energy level is given by

$$x_1 = \frac{N!}{n_1!(N-n_1)!}$$

For the second state E_2 only $(N-n_1)$ particles are available and n_2 particles are in state E_2. The no. of independent ways of choosing n_2 particles for the second energy level is given by

$$x_2 = \frac{(N-n_1)!}{n_2!(N-n_1-n_2)!}$$

similarly for n_k particles

$$x_k = \frac{(N-n_1-n_2\cdots n_{k-1})!}{n_k!(N-n_1-n_2-n_k)!}$$

Since these numbers are independent, the number of distinct ways of obtaining the distribution of the particles among the energy levels is given by

$$x = x_1 x_2 \cdots x_i \cdots x_k$$

$$= \frac{N!}{n_1!\, n_2!\, n_3! \cdots n_k!} \text{ ——— } (5)$$

Now we have to find the number of independent ways in which the particles in the various energy levels can be distributed among the quantum states in the respective levels. The i^{th} energy level contains n_i particles, there are g_i quantum states. Each of the n_i particles can be in any one of the quantum states. Thus, there are g_i independent choices for the first particle to go in any one of the quantum state, and so on as there is no restriction of no. of particles to be occupied by any quantum state.

Hence there are $g_i^{n_i}$ ways arranging n_i particles among g_i quantum states available in the i^{th} level. Each of these arrangements is seperate independant way of distributing the particles among the quantum states. Thus, all the quantum states belonging to all the energy levels are considered, the total number of ways of arranging all the particles in the given distribution is given by

$$W = \frac{N!}{n_1!\, n_2! \cdots n_k!} \times g_1^{n_1} \times g_2^{n_2} \times \cdots \times g_k^{n_k}$$

$$= N! \prod_{i=1}^{k} \left[\frac{g_i^{n_i}}{n_i!} \right] \text{ ——— } (6)$$

where the symbol $\prod$ represents the product of quantities $\frac{g_i^{n_i}}{n_i!}$ for all values of i from 1 to k.

The most probable distribution of the particles among the energy states in equilibrium is that for which the probability of occurance is maximum, i.e for which W is maximum. For mathematical convenience, we consider the condition for maximum value of log w. The condition for maximum value of log w is

$$d(\log w) = 0 \quad \text{———} (7)$$

Taking log of eq^n (6), we have

$$\log w = \log N! + \log\left[\prod_i \left(\frac{g_i^{n_i}}{n_i!}\right)\right]$$

$$= \log N! + \sum_{i=1}^{k} (n_i \log g_i - \log n_i!) \quad \text{———} (8)$$

Since the number of particles is very large, N! and $n_i!$ are also very large. Using sterling's formula

$$\boxed{\log x! = x \log x - x}$$

we get

$$\log N! = N \log N - N \quad \text{———} (9)$$

and

$$\log n_i! = n_i \log n_i - n_i \quad \text{———} (10)$$

$$\therefore \log w = N \log N - N + \sum_{i=1}^{k} (n_i \log g_i - n_i \log n_i + n_i) \quad \text{———} (11)$$

Differentiating above eq^n (11)

$$\frac{d(\log w)}{dn_i} = 0 + \sum\left[n_i \cdot 0 + \log g_i \, dn_i - n_i \frac{\cancel{n_i}}{n_i} - \log n_i \, dn_i + 1\right]$$

$$= \sum\left[0 + \log g_i \, dn_i - 1 - \log n_i \, dn_i + 1\right]$$

$$= \sum\left[\log g_i - \log n_i\right] dn_i$$

$$= \sum\left[-\log\left(\frac{n_i}{g_i}\right)\right] dn_i$$

For maximum probability $d(\log w) = 0$, so,

$$\sum -\log\left(\frac{n_i}{g_i}\right) dn_i = 0 \qquad (12)$$

Let α and β be the Langrange's undetermined multipliers independent of n_i's so we have,

$$\sum\left[-\log\left(\frac{n_i}{g_i}\right) - \alpha - \beta E_i\right] dn_i = 0$$

$$\therefore \quad -\log\left(\frac{n_i}{g_i}\right) - \alpha - \beta E_i = 0$$

$$\therefore \quad \log\left(\frac{n_i}{g_i}\right) = -\alpha - \beta E_i$$

or $$\frac{n_i}{g_i} = e^{(-\alpha - \beta E_i)}$$

or $$n_i = g_i e^{(-\alpha - \beta E_i)}$$

$$\boxed{n_i = g_i e^{(-\alpha - \beta E_i)}} \qquad (13)$$

This equation is known as the Maxwell-Boltzmann energy distribution law.

The value of undetermined multiplier β is evaluated as $\beta = \frac{1}{kT}$ (14)

$$\therefore n_i = g_i e^{-\alpha} e^{-E_i/kT} \qquad (15)$$

where k is Boltzmann constant and T is absolute temperature. The quantity $e^{-E_i/kT}$ is known as the Boltzmann factor.

* Quantum Statistics

In classical statistics, Maxwell-Boltzmann explained well on the energy and velocity distribution of the molecules of an ideal gas. It has been assumed that all the energy levels are accessible to all the particles of the system. However, there are certain energy levels prohibited to a certain groups of particles. This is the reason that M.B. statistcs failed to explain other experimentally observed phenomena such as black body radiation, photoelectric effect, specific heat at low temperatures etc. so, a new quantum concept of discrete exchange of energy between systems was developed which is known as quantum statistics.

In 1924, the quantum statistics was first formulated by Bose in the deduction of plank's radiation law. Einstein in the same year employed it in kinetic theory of gases as a substitute for the classical Boltzmann statistics, known as Bose-Einstein (B-E) statistics. Two years later, in 1926, Fermi and Dirac modified B-E. statistics in certain cases, on the basis of Pauli's exclusion principle, known as Fermi-Dirac (F-D) statistics. Thus, the quantum statistics can be put in to two sub-classes as:

<I> Bose-Einstein statistics

<II> Fermi-Dirac statistics,

depending upon symmetric or antisymmetric wave-functions.

For developing a quantum statistical theory for a system containing a large number of identical and indistinguishable particles, it is necessary to consider the wave function for many body state as

$$\psi = \psi(\bar{r}_1, \bar{r}_2, \bar{r}_3 - - - -)$$

where $\bar{r}_1, \bar{r}_2, \bar{r}_3, - - -$ are position vectors of the first, second, third, - - - particles respectively.

Symmetric Wavefunction

If on interchanging any pair of identical particles, there is no change of the ~~spin~~ sign of the wavefunction, i.e if

$$\psi(\bar{r}_1, \bar{r}_2, \bar{r}_3, - - - -) = + \psi(\bar{r}_2, \bar{r}_1, \bar{r}_3, - - - -)$$

the wave function is said to be symmetric.

Anti-symmetric Wavefunction

If on interchanging any pair of identical particles, the sign of the wavefunction changes, i.e if

$$\psi(\bar{r}_1, \bar{r}_2, \bar{r}_3, - - - - -) = - \psi(\bar{r}_2, \bar{r}_1, \bar{r}_3, - - - -)$$

the wavefunction is said to be anti-symmetric.

For the particles having spin angular momenta that are zero or integral multiples of the unit $\hbar\ (= h/2\pi)$, the wave-functions are symmetric, and for particles with spin angular momenta that are odd half-integral multiples of $\hbar$, the wave-functions are antisymmetric.

* Bose-Einstein Distribution Law

Basic Postulates

In Bose-Einstein statistics, the conditions are:

<1> The particles of the system are identical and indistinguishable.

<2> Any number of particles can occupy a single cell in the phase space.

<3> The size of the cell can not be less than h^3, where h is plank's constant ($h = 6.63 \times 10^{-34}$ J-S)

(4> The number of phase space cells is comparable with the number of particles.

(5> Bose-Einstein statistics is applicable to particles with integral spin angular momentum in units of $h/2\pi$. All the particles which obey B.E. statistics are known as Bosons.

Consider a system consisting of n independent and indistinguishable particles. Let there be $n_1, n_2, n_3, ---, n_i, --- n_k$ particles having mean energy values $E_1, E_2, ---, E_i, --- E_k$ respectively in compartments numbered as 1, 2, ---, i, ---, k containing $g_1, g_2, ---, g_i, ---, g_k$ cells respectively in them. The total number of particles in the system is

$$n = n_1 + n_2 + --- + n_i + --- + n_k \qquad \text{——— (1)}$$

Now consider i^{th} compartment. It has n_i indistinguishable particles distributed among its g_i cells. suppose, that n_i particles are arranged in a row and distributed among g_i quantum states with $(g_i - 1)$ partitions in between. The total number of possible arrangements of particles and partitions is equal to the the total number of permutations of $(n_i + g_i - 1)$ objects in a row. Therefore, the total possible ways of arranging n_i particles with g_i-1 partitions

$$= (n_i+g_i-1)! \quad \text{———} (2)$$

As the particles are indistinguishable, the rearrangement of particles among themselves will not give rise to any new distribution. The number of such meaningless permutations is $n_i!$ Hence above result would be divided by $n_i!$. Moreover, this ~~is~~ also includes permutations of (g_i-1) partitions among themselves. These permutations also donot produce different states and hence are meaningless. The above result is further divided by $(g_i-1)!$

Hence, the actual number of ways in which n_i particles are to be distributed in g_i cells in the i^{th} compartment is given by

$$W_i = \frac{(n_i+g_i-1)!}{n_i!\,(g_i-1)!} \quad \text{———} (3)$$

Similar expressions will be obtained for other compartments. Therefore, the total number of different arrangements for all the n particles of the system gives thermodynamic probability.

$$W_{(n_1,n_2\cdots n_i,\cdots n_k)} = \frac{(n_1+g_1-1)!}{n_1!\,(g_1-1)!} \times \frac{(n_2+g_2-1)!}{n_2!\,(g_2-1)!} \times \cdots \times \frac{(n_i+g_i-1)!}{n_i!\,(g_i-1)!} \times \cdots \times \frac{(n_k+g_k-1)!}{n_k!\,(g_k-1)!}$$

$$= \prod_{i=1}^{k} \frac{(n_i+g_i-1)!}{n_i!\,(g_i-1)!} \quad \text{———} (4)$$

where Π denotes multiplication of terms stated above for various values of i from 1 to k.

The most probable microstate corresponds to the state of maximum thermodynamic probability.

In the above eq^n (4), n_i and g_i both are very large numbers. Hence we may neglect 1 in the above expression.

$$\therefore \quad W = \prod \frac{(n_i+g_i)!}{n_i!\, g_i!} \qquad (5)$$

Taking natural logarithm on both sides, we have

$$\log w = \sum_{i=1}^{k} \left[\log(n_i+g_i)! - \log n_i! - \log g_i!\right]$$

As n_i and g_i are very large numbers, we can use Sterlings approximation

$$\log n! = n\log n - n \qquad (6)$$

Applying sterlings approximation, we get

$$\log w = \sum_{i=1}^{k} \left[(n_i+g_i)\log(n_i+g_i) - (n_i+g_i) - n_i\log n_i + n_i - g_i\log g_i + g_i\right]$$

$$= \sum \left[(n_i+g_i)\log(n_i+g_i) - n_i\log n_i - g_i\log g_i\right] \qquad (7)$$

Here g_i is not subject to variation where as n_i varies continuously

For the state of maximum thermodynamic probability we differentiate equation (7) and equate to zero i.e

$$d(\log w) = 0$$

$$d(\log w) = \sum \left[dn_i\log(n_i+g_i) + (n_i+g_i)\frac{1}{(n_i+g_i)}dn_i - dn_i\log n_i - n_i\frac{1}{n_i}dn_i - 0\right]$$

$$= \Sigma\, dn_i \log(n_i + g_i) - dn_i \log n_i$$

$$\therefore\; d(\log w) = \Sigma \left[\log(n_i + g_i) - \log n_i\right] dn_i$$

$$= \Sigma \left[\log \frac{(n_i + g_i)}{n_i}\right] dn_i$$

$$\therefore\; -\Sigma \left[\log \frac{n_i}{(n_i + g_i)}\right] dn_i = 0$$

or $$\Sigma \left[\log \frac{n_i}{(n_i + g_i)}\right] dn_i = 0 \quad \text{——— } (8)$$

Applying ~~Lag~~ Langrangian method of undetermined multiplyers α and β for the above eqn.

$$\therefore\; \Sigma \left[\log\left(\frac{n_i}{n_i + g_i}\right) + \alpha + \beta E_i\right] dn_i = 0 \quad \text{——— } (9)$$

The variations dn_i are independent of eachother, Hence we get

$$\log\left(\frac{n_i}{n_i + g_i}\right) + \alpha + \beta E_i = 0$$

$$\therefore\; \log\left(\frac{n_i}{n_i + g_i}\right) = -\alpha - \beta E_i$$

or $$\frac{n_i}{n_i + g_i} = e^{-\alpha - \beta E_i} = e^{-(\alpha + \beta E_i)}$$

or $$\frac{n_i + g_i}{n_i} = e^{\alpha + \beta E_i}$$

or $$1 + \frac{g_i}{n_i} = e^{\alpha + \beta E_i}$$

or $$\frac{g_i}{n_i} = e^{\alpha + \beta E_i} - 1$$

or $$g_i = \left(e^{\alpha + \beta E_i} - 1\right) \cdot n_i$$

or $$\boxed{n_i = \frac{g_i}{e^{\alpha+\beta E_i} - 1}} \qquad (10)$$

This is known as Bose-Einstein's Distribution Law for an assembly of bosons.

* Fermi-Dirac Distribution Law

Consider a system consisting of n independent and indistinguishable particles. The particles have definite energies and occupy definite positions.

Let the compartments be marked 1, 2, ---, i, ---k and their mean energy values be represented by $E_1, E_2, --- E_i, ---, E_k$ containing $g_1, g_2, ---, g_i, ---, g_k$ cells respectively. The total number of particles in the system is

$$n = n_1 + n_2 + ---- + n_i + --- + n_k \qquad (1)$$

Basic Postulates

<1> The particles are indistinguishable so that there is no distinction between the various ways in which n_i particles chosen.

<2> The particles obey Pauli's exclusion principle according to which there can be either no particle or only one particle in a given cell. Therefore, the number of cells must be much greater than the number of particles, i.e. $g_i >> n_i$

Out of n_i number of particles in the i^{th} compartment with energy E_i, the first particle can be placed in any one of the available g_i states. Thus, the first particle can be distributed in g_i different ways then (g_i-1) states will remain vacant. The second particle can be arranged in (g_i-1) different ways and the process continues.

Thus, the total number of different ways of arranging n_i particles among available g_i states with energy level E_i is given by

$$= g_i(g_i-1)(g_i-2)(g_i-3)\cdots[g_i-(n_i-1)]$$

$$= \frac{g_i}{(g_i-n_i)!} \quad \text{———} \quad (2)$$

Since the particles are indistinguishable, it would not be possible to detect any difference when n_i particles are reshuffled into different ways occupied by them in the energy level E_i. Thus out of these n_i permutations will be meaningless. Therefore, the total number of different and distinguishable ways is

$$= \frac{g_i!}{n_i!(g_i-n_i)!} \quad \text{———} \quad (3)$$

So, the thermodynamic probability for the microstates $(n_1, n_2, \ldots, n_i, \ldots, n_k)$ of the system is given by

$$W_{(n_1,n_2,\ldots,n_i,\ldots n_k)} = \frac{g_1!}{n_1!(g_1-n_1)!} \times \frac{g_2!}{n_2!(g_2-n_2)!} \times \cdots \times \frac{g_i!}{n_i!(g_i-n_i)}$$

$$\times \cdots \times \frac{g_k!}{n_k!(g_k-n_k)!}$$

$$\omega = \prod_{i=1}^{k} \frac{g_i!}{n_i!(g_i-n_i)!} \quad \text{———} \quad (4)$$

where Π denotes multiplication of terms for various values of i from 1 to k.

Taking natural logarithm on both sides of eq^n(4)

$$\log \omega = \sum_{i=1}^{k} \left[\log g_i! - \log n_i! - \log(g_i-n_i)!\right] \quad \text{———} \quad (5)$$

As n_i and g_i are very large we can use Sterling approximation

$$\log n! = n \log n - n \quad \text{———} \quad (6)$$

we have

$$\log \omega = \sum_{i=1}^{k} \left[g_i \log g_i - g_i - n_i \log n_i + n_i - (g_i - n_i)\log(g_i - n_i) + (g_i - n_i) \right]$$

$$= \sum \left[g_i \log g_i - n_i \log n_i - (g_i - n_i)\log(g_i - n_i) \right]$$

Here g_i is not subject to variation but n_i varies continuously. Differentiating both sides, we have

$$d(\log \omega) = \sum \left[0 - n_i \frac{1}{n_i} dn_i - dn_i \log n_i + (g_i - n_i) \frac{1}{(g_i - n_i)} dn_i + \log(g_i - n_i) dn_i \right]$$

$$= \sum \left[-dn_i - \log n_i \, dn_i + dn_i + \log(g_i - n_i) dn_i \right]$$

$$d(\log \omega) = \sum \left[\log(g_i - n_i) - \log n_i \right] dn_i \quad \text{———— (7)}$$

For the state of maximum thermodynamic probability

$$d(\log \omega) = 0$$

$$\therefore \sum_{i=1}^{k} \left[\log(g_i - n_i) - \log n_i \right] dn_i = 0$$

or

$$\sum \left[\log \frac{g_i - n_i}{n_i} \right] dn_i = 0$$

or

$$-\sum \left[\log \frac{n_i}{g_i - n_i} \right] dn_i = 0$$

or

$$\sum \left[\log \frac{n_i}{g_i - n_i} \right] dn_i = 0 \quad \text{———— (8)}$$

Using Lagrangian method of undetermined multipliers α and β we have

$$\sum \left[\log \left(\frac{n_i}{g_i - n_i} \right) + \alpha + \beta E_i \right] dn_i = 0 \quad \text{———— (9)}$$

As the variations dn_i are independent of each other, we get

$$\log\left(\frac{n_i}{g_i-n_i}\right)+\alpha+\beta E_i = 0$$

or $$\log\frac{n_i}{(g_i-n_i)} = -(\alpha+\beta E_i)$$

$$\therefore \frac{n_i}{g_i-n_i} = e^{-(\alpha+\beta E_i)}$$

$$\therefore \frac{g_i-n_i}{n_i} = e^{(\alpha+\beta E_i)}$$

$$\frac{g_i}{n_i}-1 = e^{(\alpha+\beta E_i)}$$

$$\frac{g_i}{n_i} = e^{(\alpha+\beta E_i)}+1$$

$$\therefore g_i = \left[e^{(\alpha+\beta E_i)}+1\right].n_i$$

$$\boxed{n_i = \frac{g_i}{e^{(\alpha+\beta E_i)}+1}} \quad (10)$$

This equation represent the most probable distribution of the particles among various energy levels for a system obeying Fermi-Dirac statistics and is therefore known as Fermi-Dirac distribution law, for an assembly of Fermions.

The parameter $\beta = \frac{1}{kT}$

where k is Boltzmann's constant.

* Comparision of M.B., B.E. and F.D. statistics -

	Maxwell-Boltzmann	Bose-Einstein	Fermi-Dirac
1	particles are distinguishable	particles are indistinguishable	particles are indistinguishable
2	only particles are taken into consideration	only quantum states are taken into consideration	only quantum states are taken into consideration
3	There is no restriction on number of particles in a given state	There is no restriction on number of particles in a given state.	There is restriction on number of particles in a given state
4	Phase space volume is not known	Phase space is known $V = h^3$	Phase space is known $V = h^3$
5	Maximum probability distribution $\propto \frac{1}{e^{(\alpha+\beta E_i)}}$	Maximum probability distribution $\propto \frac{1}{[e^{(\alpha+\beta E_i)} - 1]}$	Maximum probability distribution $\propto \frac{1}{[e^{(\alpha+\beta E_i)} + 1]}$
6	Applicable to ideal gas molecules	Applicable to photons and symmetrical particles known as bosons.	Applicable to electrons and antisymmetric particles known as. fermions
7	At absolute zero temperature energy is taken to be. zero	The energy at absolute zero is taken to be zero	The energy at absolute zero is not taken to be zero
8	$n_i = \frac{g_i}{e^{(\alpha+\beta E_i)}}$	$n_i = \frac{g_i}{[e^{(\alpha+\beta E_i)} - 1]}$	$n_i = \frac{g_i}{[e^{(\alpha+\beta E_i)} + 1]}$
9	—	At high temperature Bose-Einstein distribution approaches Maxwell-Boltzmann distribution	At high temperatures Fermi-Dirac distribution approaches Maxwell-Boltzmann distribution.
10	No. of distinguishable ways are given by $W = \prod \frac{g_i^{n_i}}{n_i!}$	No. of distinguishable ways are given by $W = \prod \frac{(n_i + g_i - 1)!}{n_i!\,(g_i - 1)!}$	No. of distinguishable ways are given by $W = \prod \frac{g_i!}{n_i!\,(g_i - n_i)!}$

* Photon gas

Consider a double walled hollow sphere having a very narrow opening at one point is a perfect absorber as its inner surface is coated with lamp black and a sharp projection opposite to the opening. If the radiation enters through the narrow opening, it is absorbed completely inside by successive reflections on the inner walls, as designed by Fery. If the encloser is maintained at a constant temperature T, then atoms of the walls of the encloser emit electromagnetic radiation and at the same time these radiations are absorbed by the atoms in walls. Thus, the atoms of the walls will emit and reabsorb photons continuously. After certain time, a situation will be established when the emission and absorbtion of photons attains thermodynamic equilibrium. At this stage, the amount of energy emitted by the atoms per unit time is equal to the amount of energy absorbed by atoms per unit time.

Thus, the interaction of electromagnetic radiation with matter led to the idea that electromagnetic radiation are composed of discrete energy particles called photons. Each po photon has an energy $h\nu$ and momentum h/λ. Hence ν is frequency and λ is the wavelength of the radiation. The electromagnetic radiation trapped in a cavity and in thermal equilibrium with the walls of the cavity are termed as black body radiation. In the equilibrium condition, the black body radiation can be considered as the photon gas.

* Electron Gas

Metals are very good conductors. The high conductivity of metals is due to the presence of free electrons. These free electrons inside a metallic conductor move freely. They continuously collide with the fixed atoms and thus behave like an electron gas.

Consider an electron gas having n free electrons in a metallic conductor whose volume is V. Let the total energy of electron ~~by~~ be E. Practically, at all the temperatures, the lower level energy band are filled with electrons. The upper level energy bands are only partially filled with electrons. The distribution of electrons is to be considered only in the upper bands called the conduction band.

As the energy of the electrons in the conduction band is continuous, the term g_i replaced by $g(E)dE$, ~~electrons~~ Hence $n(E)dE$ electrons have energy in the range E and E+dE. According to Fermi-Dirac distribution law.

$$n_i = \frac{g_i}{e^{\alpha+\beta E_i}+1} \quad \text{———} \quad (1)$$

put $g_i = g(E)dE$ and $n_i = n(E)dE$ in above eqⁿ

$$n(E)dE = \frac{g(E)dE}{e^{\alpha+\beta E_i}+1} \quad \text{———} \quad (2)$$

As electron has spin $\pm 1/2$, the total number of states in the sphere is twice. The quantity $V/(2\pi)^3$ refers to the translational states per unit volume in the Fermi space. Fermi sphere of radius k_F has the total number of particles accomodated

$$n(E) = \frac{2\left[\frac{V}{(2\pi)^3}\right]\left(\frac{4}{3}\pi k_F^3\right)}{e^{(\alpha+\beta E_i)}+1} \quad \text{———} \quad (3)$$

Here $$E = \frac{p^2}{2m} = \frac{\left(\frac{h}{2\pi}\right)^2 k_F^2}{2m}$$

$$\therefore \quad k_F^2 = \frac{p^2}{(h/2\pi)^2} \qquad \therefore \quad k_F = \frac{p}{h/2\pi}$$

$$\therefore \quad k_F = \frac{2\pi p}{h} \quad \text{———} \quad (4)$$

put eq^n (4) in eq^n (3) we get

$$n(E) = \frac{2\left[\frac{V}{(2\pi)^3}\right]\left(\frac{4}{3}\pi\left(\frac{2\pi p}{h}\right)^3\right)}{e^{(\alpha+\beta E_i)}+1}$$

But $p = (2mE)^{1/2}$

$$n(E) = \frac{2\left[\frac{V}{(2\pi)^3}\right]\left[\frac{4}{3}\pi\left(\frac{2\pi}{h}\right)^3 \cdot (2mE)^{3/2}\right]}{e^{(\alpha+\beta E_i)}+1}$$

$$n(E) = \frac{\left[\frac{8\pi V}{(3h)^3}\right](2m)(2m)^{1/2} E^{3/2}}{e^{(\alpha+\beta E_i)}+1}$$

Differentiating

$$n(E)dE = \frac{\left[\frac{8\pi V}{(3h)^3}\right](2m)(2m)^{1/2} \times \frac{3}{2} E^{1/2} dE}{e^{(\alpha+\beta E_i)}+1}$$

$$n(E)dE = \frac{\left[\frac{8\pi V (2m^3)^{1/2}}{h^3}\right] E^{1/2}}{e^{\alpha+\beta E_i}+1} \quad \text{———} \quad (5)$$

Equation (5) represents the distribution of energy among free electrons..